Photonuclear Reactions induced by Intense Short Laser Pulses

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Abstract

A measurement of the decay in time of nuclei excited by an intense short laser pulse of energy E_0 yields the Fourier transform of the autocorrelation function of the associated scattering matrix. We determine the optimal length (in time) of the pulse and evaluate the time–decay function using random–matrix theory. That function is shown to contain information not otherwise available. We approximate that function in a manner that is useful for the analysis of data. For E_0 below the threshold energy E_n of the first neutron channel, the time–decay function is exponential in time t while it is the product of an exponential and a power in t for $E_0 > E_n$. The comparison of the measured decay functions in both energy domains yields an unambiguous and novel test of random–matrix theory in nuclei.

1. Purpose

ELI, the "Extreme Light Infrastructure", an ambitious European project to generate laser beams of extremely high intensity, is close to construction. Parallel to that development, experiments that will use those beams are being planned, and theorists are called upon to develop the concepts and tools needed for their analysis. At the workshop on ELI held in Palaiseau (France) April 27/28, 2009, photonuclear reactions induced by an intense laser pulse of high energy (several MeV) received much attention, see Ref. [1]. The required high–energetic directed pulsed gamma rays are supposed to be produced by Compton backscattering of a short laser pulse with much lower energy on a sheet of electrons ejected from a thin foil hit by the ELI pulse.

In this paper, we present a theoretical study of nuclear reactions induced by short laser pulses of several MeV energy. In particular, we address the following questions: (i) Which observable is measured in a photonuclear reaction induced by a very short laser pulse? (ii) Which novel information is provided by data of that type? (iii) What is the optimal length in time of a short laser pulse for such a reaction?

We focus attention on the main mode of nuclear excitation by gamma quanta of several MeV energy, the electric dipole mode. For a target nucleus in its ground state with spin J and parity π, dipole absorption leads to excited states with spin $J \pm 1$ and opposite parity $-\pi$. The analysis of experimental data will be simplest when the spins of the excited states are uniquely defined. Therefore, we consider an even–even target nucleus with a ground state of spin zero and positive parity. Then the states excited by dipole absorption have spin 1 and negative parity. Depending on excitation energy, these may lie below or above the first particle threshold. That is typically the threshold for neutron emission. In medium–weight and heavy nuclei, that threshold has an excitation energy of 5 to 8 MeV. Nuclear states right above neutron threshold have been studied by time–of–flight spectroscopy in slow neutron scattering, mostly on even–even target nuclei. Such states appear as isolated s–wave resonances with spin 1/2 and positive parity, with a typical spacing of 10 eV and a typical width of 1 eV. The statistical analysis of such resonances shows that the spacings and widths follow the predictions of random–matrix theory (RMT), see the review [2]. In recent years, states in even–even nuclei with spin 1 and negative parity close to the first particle threshold forming the "pygmy dipole resonance" have been intensely studied experimentally, mainly with the help of the resonance fluorescence technique, see Refs. [3, 4] and references therein. In Ref. [4] deviations from RMT predictions were found that have so far not been fully explained theoretically. Our work is based on the assumption that the states excited by an intense short laser pulse are governed by RMT. We explore the consequences of that assumption and propose an experimental test for it.

For reasons given below, we consider a laser pulse of 10^{-19} to 10^{-20} s duration and a mean energy of several MeV. The pulse coherently excites a band of 1^- states. The band width is several 10 keV, the number of 1^- states involved is typically 10^3 to 10^4. We refer to these states as to compound–nucleus (CN) resonances. These will subsequently decay. CN resonances below neutron threshold decay by gamma emission, those above neutron threshold preferentially by neutron emission. Gamma emission is possible but less likely. In both cases the detection of the emitted particle with highest energy unambiguously iden-

tifies decay into the ground state of the residual nucleus. We do not focus attention solely on that decay mode since in the case of gamma decay, the intensity may be too low, and a summation of intensities corresponding to several or many gamma decay modes leaving the target nucleus in its ground or one of its excited states, may be called for. In any case we deal with a two–channel situation. The incident channel γ_0 is defined by the target nucleus in its ground state plus a dipole gamma quantum, the exit channel b by the emitted particle ($b = n$ for neutron emission leaving the residual nucleus in its ground state, $b = \gamma_i$ with $i = 0, 1, \ldots, \Lambda$ for gamma emission leaving the residual nucleus in its ground or any of Λ excited states).

Formation and decay of the CN resonances being coherent processes, it is not possible to identify any particular CN resonance as the source of the emitted particle. Interest in the data rather focuses on the time dependence of the decay. We answer the questions raised in the second paragraph by identifying the relevant observable and defining the optimal length in time of the laser pulse. We also work out the expected form of the time–decay function. We do so by using results of the analytical approach to CN reactions developed in Ref. [5].

Coherent resonance formation was previously addressed, see Ref. [6] and references therein, the decay in time of CN resonances in Refs. [7, 8, 9]. The deviations from the exponential decay in time predicted in Refs. [7, 8, 9] have been observed in microwave billiards by Fourier–transforming the measured elements of the scattering matrix, see Refs. [10, 11], and the comprehensive summary paper [12]. Here we propose a direct measurement of the time–decay function in nuclei. We go beyond Refs. [7, 8, 9] in addressing specifically the case of photonuclear reactions induced by an intense laser pulse. In Ref. [13] the autocorrelation function of the total photodissociation cross section for a chaotic atom or molecule was studied. With the help of the optical theorem, that function is related to a two–point function similar in form to the expressions studied below. The approach was extended and generalized in Refs. [14, 15].

2. Observable

To describe the effect of a short light pulse, we consider the scattering wave function $\Psi_{\gamma_0}^+(E)$. That function describes all reactions caused by a gamma quantum of energy E with wave number $k(E)$ incident on the target nucleus. Asymptotically, $\Psi_{\gamma_0}^+(E)$ has an incident wave with unit flux in channel γ_0 and outgoing waves in all channels. The amplitude of the outgoing flux in channel b is given by the element $S_{b\gamma_0}(E)$ of the scattering matrix, and the cross section $\sigma_{b\gamma_0}(E)$ feeding final channel b is given by

$$\sigma_{b\gamma_0}(E) = \frac{\pi}{k^2}|S_{b\gamma_0}(E) - \delta_{b\gamma_0}|^2 \,. \tag{1}$$

A short light pulse is described as a superposition

$$\int \mathrm{d}E \; g(E) \exp\{-iEt/\hbar\}\Psi_{\gamma_0}^+(E)$$

of scattering wave functions with different energies where t denotes the time. The envelope function $g(E)$ is a smooth function of energy E centered at energy E_0 (the mean energy of the laser pulse), has band width ΔE, and $|g(E)|^2$ is normalized to unity. The amplitude of the outgoing flux in channel b is $\int \mathrm{d}E \; g(E) \exp\{-iEt/\hbar\}S_{b\gamma_0}(E)$. The total scattered flux $I_{b\gamma_0}(t)$ in channel b versus time is given by

$$\frac{I_{b\gamma_0}(t)}{\pi} = \left| \int \mathrm{d}E \; \frac{g(E)}{k(E)} \exp\{-iEt/\hbar\}[S_{b\gamma_0}(E) - \delta_{b\gamma_0}] \right|^2 \,. \tag{2}$$

Indeed, integrating the total scattered flux over time and using Eq. (2) we obtain the energy–averaged cross section, $\int \mathrm{d}t/\hbar \; I_{b\gamma_0}(t) = \int \mathrm{d}E \; |g(E)|^2\sigma_{b\gamma_0}(E)$. In other words, $I_{b\gamma_0}(t)$ gives the decomposition in time of the cross section induced by a laser pulse of band width ΔE and time length $\Delta t = \hbar/\Delta E$. A short laser pulse is defined to have a band width ΔE that is large compared to the average spacing d of the CN resonances. We focus attention on times t that are large compared to Δt and on the associated long–time behavior $I_{b\gamma_0}^{\mathrm{long}}(t)$ of $I_{b\gamma_0}(t)$. We do so because it may be difficult to separate experimentally short–time contributions from the original laser signal, and we accordingly expect that the first round of experiments will focus on the long–time aspects of the reaction. The terms in Eq. (2) that involve $\delta_{b\gamma_0}$ are proportional to the Fourier transform of $g(E)$, have time length Δt, and are, therefore, neglegted. Thus,

$$I_{b\gamma_0}^{\mathrm{long}}(t) \tag{3}$$
$$= \frac{\pi}{k_0^2}\left| \int \mathrm{d}E \; g(E) \exp\{-iEt/\hbar\}S_{b\gamma_0}(E) \right|^2$$
$$= \frac{\pi}{k_0^2} \int \mathrm{d}E_1 \int \mathrm{d}E_2 \; g(E_1)g^*(E_2) \exp\{i(E_2 - E_1)t/\hbar\}$$
$$\times S_{b\gamma_0}(E_1)S_{b\gamma_0}^*(E_2) \,.$$

Here and in what follows we use that $k(E)$ changes little over the energy interval ΔE and replace $k(E)$ by $k_0 = k(E_0)$. (We note that I^{long} still contains a short–time component, see Eq. (14) below. That component will also be suppressed eventually). We relate $I_{b\gamma_0}^{\mathrm{long}}(t)$ to the S–matrix autocorrelation function $C_{b\gamma_0}(\varepsilon)$ defined by

$$C_{b\gamma_0}(\varepsilon) = \int \mathrm{d}E \; |f(E)|^2 S_{b\gamma_0}(E - \varepsilon/2)S_{b\gamma_0}^*(E + \varepsilon/2) \,. \tag{4}$$

The average is performed with a smooth function $|f(E)|^2$ normalized to unity the range of which extends to infinity. An example is a normalized Lorentzian with a width that is very large compared to d. We deal with isolated CN resonances for which the average total resonance width Γ obeys $\Gamma < d$. Then the function $C_{b\gamma_0}(\varepsilon)$ decreases rapidly

towards zero for $|\varepsilon| > d$. In Eq. (3) we write $E_1 = E - \varepsilon/2$, $E_2 = E + \varepsilon/2$. We use that $g(E)$ is smooth over the correlation width of $C_{b\gamma_0}(\varepsilon)$. Then $g(E \pm \varepsilon/2) \approx g(E)$, and Eq. (3) becomes

$$I_{b\gamma_0}^{\mathrm{long}}(t) \qquad\qquad\qquad\qquad\qquad (5)$$
$$\approx \frac{\pi}{k_0^2} \int \mathrm{d}\varepsilon \, \exp\{i\varepsilon t/\hbar\}$$
$$\times \int \mathrm{d}E \, |g(E)|^2 S_{b\gamma_0}(E - \varepsilon/2) S_{b\gamma_0}^*(E + \varepsilon/2)$$
$$\approx \frac{\pi}{k_0^2} \int \mathrm{d}\varepsilon \, \exp\{i\varepsilon t/\hbar\} C_{b\gamma_0}(\varepsilon) \,.$$

The approximation leading to Eq. (5) requires $\Delta E \gg d$. Otherwise, finite–range–of–data errors have to be taken into account. If that condition is met, the long–time part of the signal $I_{b\gamma_0}(t)$ measures the Fourier transform of the S–matrix autocorrelation function $C_{b\gamma_0}(\varepsilon)$. The inequality $\Delta E \gg d$ yields the first constraint on the length in time of the laser pulse: Δt must be small compared to the Heisenberg time $\hbar/d$.

A second constraint arises because dipole absorption in nuclei is governed by the giant dipole resonance. Depending on mass number the resonance occurs at excitation energies between 10 and 15 MeV and has a width Γ^{dip} of several MeV. The resonance causes a secular variation of the average dipole strength of the CN resonances. For a clean theoretical analysis, it is desirable to separate that secular variation from the average over resonances taken in the first of Eqs. (5). That condition is met if $\Delta E \ll \Gamma^{\mathrm{dip}}$.

Combining the two constraints we find $d \ll \Delta E \ll \Gamma^{\mathrm{dip}}$. With $d \approx 10$ eV and $\Gamma^{\mathrm{dip}} \approx$ several MeV, the constraints are met for ΔE in the range 10 keV to 100 keV, depending on excitation energy and mass number. That corresponds to a time length $\Delta t \approx 0.5 \times 10^{-19}$ s to 0.5×10^{-20} s. Under these conditions, irradiation of a target with an intense laser pulse yields information on the decay in time of the CN resonances. As shown in Refs. [8, 9], that decay is not expected to be exponential in general. It would certainly be exciting to measure directly the time–dependence of the decay of the CN, i.e., of the signal $I_{b\gamma_0}(t)$, see Section 3, even though measurements in microwave billiards have already provided clear evidence for non–exponential decay in these systems [10, 11, 12]. Such measurements are possible because the average width for neutron decay is less than 1 eV and that for gamma decay even smaller. The overall decay time of the CN resonances is, thus, much longer than the duration of the incident laser pulse, and the two signals are clearly separated.

We note that a band width $\Delta E \approx 10 \ldots 100$ keV also guarantees that in exciting CN resonances right above neutron threshold, one avoids excitation energies where neutron emission leaving the residual nucleus in an excited state becomes possible. (The excitation energy of the first excited state in the residual nucleus is typically 100 to 200 keV). This is desirable as otherwise in the expression for $C_{b\gamma_0}(\varepsilon)$ the additional neutron channel must be taken into account, even if only decay into the ground state is experimentally measured.

The decay in time of CN resonances excited by a short and intense laser pulse is described by the Fourier transform of the S–matrix autocorrelation function $C_{b\gamma_0}(\varepsilon)$. Is that information novel, or are other data available or within experimental reach that would yield the same information? An obvious possibility would be, for instance, a measurement of the (γ_0, n) reaction cross section $\sigma_{n,\gamma_0}(E)$ from which the cross–section autocorrelation function

$$C_{n,\gamma_0}^\sigma(\varepsilon) = \int \mathrm{d}E \, |f(E)|^2 \sigma_{n,\gamma_0}(E + \varepsilon/2) \sigma_{n,\gamma_0}(E - \varepsilon/2) \,,$$

with $f(E)$ as defined in Eq. (4) can be obtained. For isolated resonances, $C_{n,\gamma_0}^\sigma(\varepsilon)$ is neither theoretically accessible (in contrast to $C_{b\gamma_0}(\varepsilon)$, see Section 3), nor is it simply related to $C_{b\gamma_0}(\varepsilon)$, see Ref. [16]. Generally speaking, the coherent decay in time of the CN resonances is described by an amplitude correlation function. An intensity autocorrelation function such as $C_{n,\gamma_0}^\sigma(\varepsilon)$ does not provide equivalent information (except in the Ericson regime where $\Gamma \gg d$). Thus, the information available from $C_{b\gamma_0}(\varepsilon)$ is unique.

3. S–Matrix Autocorrelation Function

The autocorrelation function for CN scattering has been calculated analytically for CN resonances that obey RMT statistics [5]. In Ref. [17] and references therein it was shown that this function correctly describes chaotic scattering. Without going into details, we summarize some salient features of that approach, see Refs. [11, 12], and cite the result. The S-matrix with elements $S_{ba}(E)$ is decomposed into an average part $\langle S_{ba} \rangle$ and a fluctuating part $S_{ba}^{\mathrm{fl}}(E)$,

$$S_{ba}(E) = \langle S_{ba} \rangle + S_{ba}^{\mathrm{fl}}(E) \,. \qquad (6)$$

The average is taken over the ensemble of random matrices. In what follows we use the equality of ensemble average and energy average which holds provided the latter is performed over an energy interval containing a large number of CN resonances with a smooth averaging function $|f(E)|^2$ as in Eq. (4). The average S–matrix $\langle S_{ba} \rangle$ describes the fast part of the reaction. Simple models like the optical model for elastic scattering or direct–reaction models yield reliable theoretical results for $\langle S_{ba} \rangle$. The fluctuating part $S_{ba}^{\mathrm{fl}}(E)$ describes the slow part of the reaction, i.e., formation and subsequent decay of the CN resonances. Because of the complexity of the latter, the precise energy dependence of $S_{ba}^{\mathrm{fl}}(E)$ cannot be predicted theoretically. Using a random–matrix model for the CN resonances one can, however, calculate the energy–autocorrelation function of $S_{ba}^{\mathrm{fl}}(E)$. That function is given in terms of the average spacing d of the CN resonances and of the elements of the average S–matrix $\langle S \rangle$ which serve as input parameters. We first assume that $\langle S \rangle$ is diagonal and later discuss

modifications due to the non–zero non–diagonal elements. We have [5]

$$\langle S_{ba}^{\mathrm{fl}}(E_1)(S_{ba}^{\mathrm{fl}}(E_2))^*\rangle = \langle S_{ba}^{\mathrm{fl}}(E - \varepsilon/2)(S_{ba}^{\mathrm{fl}}(E + \varepsilon/2))^*\rangle$$

$$= \prod_{i=1}^{2}\int_0^{+\infty}\mathrm{d}\lambda_i\int_0^1\mathrm{d}\lambda\; J_{ba}(\lambda_1,\lambda_2,\lambda)$$

$$\times \frac{1}{8}\mu(\lambda_1,\lambda_2,\lambda)\;\exp\left\{-\frac{i\pi\varepsilon}{d}(\lambda_1+\lambda_2+2\lambda)\right\}$$

$$\times \prod_e \frac{(1-T_e\lambda)}{(1+T_e\lambda_1)^{1/2}(1+T_e\lambda_2)^{1/2}}\,. \tag{7}$$

We note that the autocorrelation function (7) depends only on the energy difference $\varepsilon = E_2 - E_1$. The factor $\mu(\lambda_1,\lambda_2,\lambda)$ is an integration measure and is given by

$$\mu(\lambda_1,\lambda_2,\lambda) = \frac{(1-\lambda)\lambda|\lambda_1-\lambda_2|}{\prod_{i=1}^2[((1+\lambda_i)\lambda_i)^{1/2}(\lambda+\lambda_i)^2]}\,, \tag{8}$$

while

$$J_{ba}(\lambda_1,\lambda_2,\lambda) = (1+\delta_{ab})T_aT_b \tag{9}$$

$$\times\left(\sum_{i=1}^2\frac{\lambda_i(1+\lambda_i)}{(1+T_a\lambda_i)(1+T_b\lambda_i)} + \frac{2\lambda(1-\lambda)}{(1-T_a\lambda)(1-T_b\lambda)}\right)$$

$$+\;\delta_{ab}T_a^2(1-T_a)\left(\sum_{i=1}^2\frac{\lambda_i}{1+T_a\lambda_i}+\frac{2\lambda}{1-T_a\lambda}\right)^2$$

describes the dependence of the correlation function on entrance and exit channels a and b. The product in Eq. (7) extends over all open channels e. The correlation function (7) depends on the average level spacing d of the CN resonances and on the "transmission coefficients"

$$T_a = 1 - |\langle S_{aa}\rangle|^2\,. \tag{10}$$

Given $\langle S_{aa}\rangle$ and d, the right–hand side of Eq. (7) is completely known. The full S–matrix autocorrelation function reads then as

$$C_{ba}(\varepsilon) = \delta_{ab}|\langle S_{aa}\rangle|^2 + \langle S_{ba}^{\mathrm{fl}}(E_1)(S_{ba}^{\mathrm{fl}}(E_2))^*\rangle\,. \tag{11}$$

We turn to the case where $\langle S\rangle$ is not diagonal. This is of practical interest because the dipole operator gives rise to direct (γ_i,n) reactions and, thus, to non–vanishing elements $\langle S_{n\gamma_i}\rangle$. We assume that $\langle S_{n\gamma_i}\rangle$ and $\langle S_{nn}\rangle$ are known and that $\langle S_{\gamma_i\gamma_j}\rangle$ is diagonal. (Non–diagonal contributions $\langle S_{\gamma_i\gamma_j}\rangle$ with $i\neq j$ would be of second order in the electromagnetic interaction and, thus, negligible). We follow the work of Ref. [18] summarized in Ref. [11]. If $\langle S_{ab}\rangle$ is not diagonal, one has to determine the unitary transformation U_{ab} that diagonalizes the transmission matrix $P_{ab} = \delta_{ab} - \sum_c\langle S_{ac}\rangle\langle S_{bc}^*\rangle$ so that $(UPU^\dagger)_{ab} = \delta_{ab}p_a$. The S–matrix transforms according to $S\to\tilde{S} = USU^T$ where $\langle\tilde{S}\rangle$ is also diagonal. The correlation function of $\tilde{S}$ is given by Eqs. (7) to (9), with all T_a replaced by p_a. Since $\langle S_{n\gamma_i}\rangle$ is governed by the electromagnetic interaction, we

have that $|\langle S_{n\gamma_i}\rangle| \ll |\langle S_{nn}\rangle|$ for all i. This allows us to use first–order perturbation theory to calculate U and the eigenvalues p_a. We find that to lowest non–vanishing order in the electromagnetic interaction, the autocorrelation function of $S = U^\dagger\tilde{S}U^*$ is equal to that of $\tilde{S}$ as given in Eqs. (7) to (9), except for the replacement

$$T_{\gamma_i} \to p_{\gamma_i} = T_{\gamma_i} - T_n|\langle S_{n\gamma_i}\rangle|^2\,. \tag{12}$$

The autocorrelation function is then

$$C_{b\gamma_0}(\varepsilon) = |\langle S_{b\gamma_0}\rangle|^2 + \langle S_{b\gamma_0}^{\mathrm{fl}}(E_1)(S_{b\gamma_0}^{\mathrm{fl}}(E_2))^*\rangle\,. \tag{13}$$

The term $\langle S_{b\gamma_0}^{\mathrm{fl}}(E_1)(S_{b\gamma_0}^{\mathrm{fl}}(E_2))^*\rangle$ is given by Eqs. (7) to (9) with the replacement Eq. (12).

We apply these results to the case of interest by choosing $a = \gamma_0$ and calculating the Fourier transform of $C_{b\gamma_0}(\varepsilon)$ and, from there, the intensity $I_{b\gamma_0}^{\mathrm{long}}(t)$. We find (see Eq. (5))

$$\frac{k_0^2}{\pi}I_{b\gamma_0}^{\mathrm{long}}(t) = 2\pi\hbar\delta(t)\delta_{b\gamma_0}|\langle S_{\gamma_0\gamma_0}\rangle|^2 \tag{14}$$

$$+\int\mathrm{d}\varepsilon\;\exp\{i\varepsilon t/\hbar\}\langle S_{b\gamma_0}^{\mathrm{fl}}(E-\varepsilon/2)(S_{b\gamma_0}^{\mathrm{fl}}(E+\varepsilon/2))^*\rangle\,.$$

The delta function in the first term on the right–hand side of Eq. (14) signals that the contribution from $\langle S\rangle$ is instantaneous in time. That would hold for an infinitely short laser pulse. For the actual laser pulse, the signal will have the same time duration Δt as the pulse itself. In any case, that term does not contribute to the long–term behavior. Thus,

$$\frac{k_0^2}{\pi}I_{b\gamma_0}^{\mathrm{long}}(t) = \mathcal{C}_{b\gamma_0}(t) \tag{15}$$

$$= \int\mathrm{d}\varepsilon\;\exp\{i\varepsilon t/\hbar\}\langle S_{b\gamma_0}^{\mathrm{fl}}(E-\varepsilon/2)(S_{b\gamma_0}^{\mathrm{fl}}(E+\varepsilon/2))^*\rangle\,.$$

The first equation defines $\mathcal{C}_{b\gamma_0}(t)$. From Eq. (7) we have

$$\mathcal{C}_{b\gamma_0}(t) \tag{16}$$

$$= 2d\prod_{i=1}^2\int_0^{+\infty}\mathrm{d}\lambda_i\int_0^1\mathrm{d}\lambda\;\delta([td/(\pi\hbar)]-[\lambda_1+\lambda_2+2\lambda])$$

$$\times \frac{1}{8}\;\mu(\lambda_1,\lambda_2,\lambda)\prod_{i=0}^{\Lambda}\frac{(1-T_{\gamma_i}\lambda)}{(1+T_{\gamma_i}\lambda_1)^{1/2}(1+T_{\gamma_i}\lambda_2)^{1/2}}$$

$$\times ([1-\delta_{n,\mathrm{open}}] + \delta_{n,\mathrm{open}}\frac{(1-T_n\lambda)}{(1+T_n\lambda_1)^{1/2}(1+T_n\lambda_2)^{1/2}})$$

$$\times J_{b\gamma_0}(\lambda_1,\lambda_2,\lambda)\,.$$

where the replacement Eq. (12) has to be made. The function $\mathcal{C}_{b\gamma_0}(t)$ gives the decay intensity of the CN resonances and is the object of central interest. The delta function under the integral in Eq. (16) shows that contributions due to the decay of the CN resonances are delayed (all three integration variables are positive). The symbol $\delta_{n,\mathrm{open}}$ is zero (unity) if the neutron channel is closed (open), respectively.

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4. Decay in Time of the CN Resonances

In this and the next Section we work out the time dependence of the time–decay function $\mathcal{C}_{b\gamma_0}$ defined in Eqs. (15) and (16), with the replacement (12), in a time domain where the signal is sufficiently strong for detection. The average correlation width Γ of the CN resonances is approximately given by the Weisskopf estimate [19], $\Gamma = (d/(2\pi)) \sum_e p_e$. Since the CN resonances are isolated, we have $\sum_{i=0}^{\Lambda} p_{\gamma_i} < 1$. The number $(\Lambda + 1)$ of open gamma channels is very large, $\Lambda \gg 1$, so that $p_{\gamma_i} \ll 1$ individually for all i. The value of the transmission coefficient p_{γ_0} can be obtained from the average total cross section for dipole absorption given by

$$\sum_b \langle \sigma_{b\gamma_0} \rangle = \frac{2\pi}{k_0^2} [1 - \Re \langle S_{\gamma_0 \gamma_0} \rangle]. \qquad (17)$$

According to the statistical model, $\langle S_{\gamma_0 \gamma_0} \rangle$ is real and, for weak coupling to the channels, positive. From Eq. (10) (with T replaced by p) we have $\langle S_{\gamma_0 \gamma_0} \rangle = \sqrt{1 - p_{\gamma_0}}$ and, for $p_{\gamma_0} \ll 1$, $\langle S_{\gamma_0 \gamma_0} \rangle \approx 1 - p_{\gamma_0}/2$. That yields

$$p_{\gamma_0} \approx \frac{k_0^2}{\pi} \sum_b \langle \sigma_{b\gamma_0} \rangle. \qquad (18)$$

For the transmission coefficient T_n, we observe that the neutron has angular momentum one (zero) if the parity of the residual nucleus is positive (negative), respectively. We use the fact that for the CN resonances seen in slow s–wave neutron scattering, the average width Γ is about 1 eV. Moreover, Γ is dominated by the neutron channel, $\Gamma \approx \Gamma_n$. With $T_n = 2\pi \Gamma_n/d$ and $d \approx 10$ eV that gives $T_n \approx 0.6$ for s–wave neutrons. The transmission coefficient for p–wave neutrons is smaller by the p–wave angular momentum barrier penetration factor kR (with R the nuclear radius and k the wave number). For a laser pulse of several 10 keV band width and a mean energy of 50 keV above neutron threshold, we have $kR \approx 0.25$, leading to $T_n \approx 0.1$ or 0.2. At an energy right above neutron threshold T_n is considerably smaller, of course. Thus, T_n is much larger than any of the p_{γ_i} and of the same order as or even larger than $\sum_i p_{\gamma_i}$. This shows that we must treat T_n and p_{γ_i} in Eq. (16) differently.

The evaluation of Eq. (16) seems to require the knowledge of all individual transmission coefficients p_{γ_i} for all photon channels. These are not known. However, Eq. (16) can be much simplified so that it depends only on the total decay width for gamma decay and on the transmission coefficients in the entrance, in the exit, and in the neutron channel.

We are guided by the following observation. In Ref. [9] it was shown (see Eq. (6.12) of that reference) that if $\sum_i T_i \ll 1$, the Fourier transform of the S–matrix autocorrelation function $\langle S_{ab}(E - \varepsilon/2) S_{ab}^*(E + \varepsilon/2) \rangle$ is given by

$$\mathcal{C}_{ba}(t) \simeq \frac{1}{[1 + T_a \frac{dt}{\pi \hbar}][1 + T_b \frac{dt}{\pi \hbar}]} \prod_c \frac{1}{\sqrt{1 + T_c \frac{dt}{\pi \hbar}}}.$$

If the number of channels is large, that expression is approximately given by $\exp[-(dt/h)(\sum_i T_i + 2T_a + 2T_b)]$, and the approximation is excellent for times $t \ll h/(dT_i)$ for all i. If all T_i are approximately equal, that condition is met for all times for which the signal is detectable. That shows that for $\Lambda \gg 1$, the non-exponential decay predicted in Ref. [9] actually becomes unobservable: Deviations from the exponential decay form occur only for times for which the signal is too small for detection. We use that fact to simplify Eq. (16). We assume that the p_{γ_i} all have similar values and use the approximation $(1 - p_{\gamma_i}\lambda)[(1 + p_{\gamma_i}\lambda_1)(1 + p_{\gamma_i}\lambda_2)]^{-1/2} \approx \exp(-[p_{\gamma_i}/2][\lambda_1 + \lambda_2 + 2\lambda])$. That implies

$$\prod_{i=0}^{\Lambda} \frac{(1 - p_{\gamma_i}\lambda)}{(1 + p_{\gamma_i}\lambda_1)^{1/2}(1 + p_{\gamma_i}\lambda_2)^{1/2}} \to \exp\{-\sum_{i=0}^{\Lambda} p_{\gamma_i} t d/h\}. \qquad (19)$$

The arrow indicates that we have used the delta function in Eq. (16). We reiterate that the approximation Eq. (19) is expected to be excellent for times that allow a detection of the signal although it may fail asymptotically $(t \to \infty)$. To interpret the right–hand side of expression Eq. (19) we use that for $T_{\gamma_i} \ll 1$ we have $T_{\gamma_i} = 2\pi \Gamma_{\gamma_i}/d$ so that $(d/h) \sum_i T_{\gamma_i} = \Gamma_\gamma/\hbar$ where Γ_{γ_i} and Γ_γ are the average partial and total widths for gamma decay of the CN resonances, respectively. That is the expected result. The replacement Eq. (12) implies that the average partial widths and the total width for gamma decay are reduced by the direct reaction. This is plausible because some decay strength is taken away by that reaction. We denote the ensuing average total width for gamma decay by $\tilde{\Gamma}_\gamma$. The corresponding increase in neutron decay width is negligibly small in comparison with T_n.

With the approximation leading to expression (19), Eq. (16) becomes

$$\int d\varepsilon \ \exp\{i\varepsilon t/\hbar\} \langle S_{b\gamma_0}^{\text{fl}}(E - \varepsilon/2)(S_{b\gamma_0}^{\text{fl}}(E + \varepsilon/2))^* \rangle$$
$$= 2d \exp\{-\tilde{\Gamma}_\gamma t/\hbar\} F_{b\gamma_0}(t) \qquad (20)$$

where

$$F_{b\gamma_0}(t) \qquad\qquad (21)$$
$$= \prod_{i=1}^{2} \int_0^{+\infty} d\lambda_i \int_0^1 d\lambda \ \delta([td/(\pi\hbar)] - [\lambda_1 + \lambda_2 + 2\lambda])$$
$$\times \frac{1}{8} \mu(\lambda_1, \lambda_2, \lambda)$$
$$\times ([1 - \delta_{n,\text{open}}] + \delta_{n,\text{open}} \frac{(1 - T_n\lambda)}{(1 + T_n\lambda_1)^{1/2}(1 + T_n\lambda_2)^{1/2}})$$
$$\times J_{b\gamma_0}(\lambda_1, \lambda_2, \lambda).$$

The time–decay function in Eq. (20) is the product of two factors. The first is an exponential and describes the decay due to all gamma transitions (except for additional contributions from the entrance and the exit channels). The second factor $F_{b\gamma_0}(t)$, given in Eq. (21), depends on the channels under consideration and on a small number

of parameters: Via Eq. (9) $F_{b\gamma_0}(t)$ depends on the transmission coefficients in the entrance and exit channels and via the delta function on the average level spacing of the CN resonances. Moreover, $F_{b\gamma_0}(t)$ depends on whether the neutron channel is open or not, and on the value of T_n. Interest focuses on $F_{b\gamma_0}(t)$ because it gives rise to observable modifications of the exponential decay.

We expect $F_{b\gamma_0}(t)$ to vanish for $t \leq 0$ (this is confirmed by the delta function in Eq. (21)), to rise to a maximum at some positive value of t, and to decay towards zero for $t \to \infty$. Obvious questions are: At which value of t does the maximum of $F_{b\gamma_0}(t)$ occur? How steep is the rise for small positive values of t? What is the form of the decay for values of t beyond the maximum? Some of these questions can be answered analytically. For small positive times, $F_{b\gamma_0}(t)$ rises quadratically. Indeed, because of the delta function in the integrand of Eq. (21) both $F_{b\gamma_0}(t)$ and its first derivative vanish at $t = 0$. For larger values of t the behavior of $F_{b\gamma_0}(t)$ is expected to differ for the three possible cases: (i) the neutron channel is closed, (ii) the neutron channel is open and neutron decay is measured ($b = n$) and (iii) the neutron channel is open but gamma decay is measured ($b = \gamma_i$). The results of Ref. [9] suggest that in case (i) the peak of $F_{\gamma_i\gamma_0}(t)$ is followed by a decay of the form t^{-2}. However, since both p_{γ_0} and p_{γ_i} are very small in comparison with $\tilde{\Gamma}_\gamma$, that decay is very slow, and the behavior of the time–decay function (20) beyond the peak of $F_{\gamma_i\gamma_0}(t)$ is governed by the first factor in Eq. (20), i.e., is purely exponential. In case (ii) we expect that $T_n d/\hbar$ is at least as large as $\tilde{\Gamma}_\gamma$. The large flux into the neutron channel should shift the peak of $F_{n\gamma_0}(t)$ towards smaller values of t than in case (i). The decay in time of $F_{n\gamma_0}(t)$ beyond its peak should be governed by the neutron channel, too, and should asymptotically be proportional to $t^{-3/2}$. The decay time is comparable with or smaller than $\hbar/\tilde{\Gamma}_\gamma$, and modifications of the exponential decay form should be detectable. Similar statements apply in case (iii) except that now beyond its peak the time decay of $F_{\gamma_i\gamma_0}(t)$ is asymptotically given by $t^{-1/2}$.

5. Numerical Results

Further insight into the time dependence of the time–decay function (16) is obtained by numerical simulation. Taking $\Lambda = 49$, choosing all p_{γ_i} equal to T, and using different sets of values for T and T_n, we have found that Eq. (20) is in all cases an excellent approximation to Eq. (16) for those values of t for which the signal is detectable, both when the neutron channel is closed and when it is open. By way of example this is shown in Figs. 1 to 4, with the black lines showing the analytic function Eq. (16) and the red crosses the approximation Eq. (20). That is an important result as it enables the analysis of data in terms of a few parameters and without knowledge of the individual values of the transmission coefficients in every gamma channel.
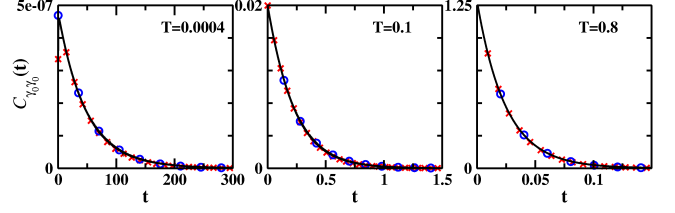

Figure 1: Intensity $\mathcal{C}_{\gamma_0\gamma_0}(t)$ versus time t in units of h/d. The neutron channel is closed, and $\Lambda = 49$ inelastic gamma channels are open, with $p_{\gamma_i} = p_{\gamma_0} = T$, $i = 1, ..., 49$ and values of the transmission coefficients chosen as indicated in the insets. Solid line (color online: black): Eq. (16). Crosses (color online: red): Approximation Eq. (20). Open circles (color online: blue): Fit of an exponential $\exp(-a_2 t)$ to the data, resulting in $a_2 \approx \sum_{i=0}^{\Lambda} p_{\gamma_i}$.

The rise in time of the time–decay function $\mathcal{C}_{b\gamma_0}(t)$ is very steep in all cases. The function reaches its maximum at times of order $\hbar/d$. The maximum value of $\mathcal{C}_{b\gamma_0}$ is mainly determined by the factor $T_a T_b$ in Eq. (9); that explains the enormous difference in scale in the figures. With increasing values of the transmission coefficients the maximum is shifted towards smaller values of t. These are very short times; it is not clear whether in the first round of experiments the decay signal can be clearly separated from the signal due to the short pulse itself (delta function in Eq. (14)). Therefore, we have focussed attention on the decay in time of $\mathcal{C}_{b\gamma_0}(t)$ beyond its maximum.
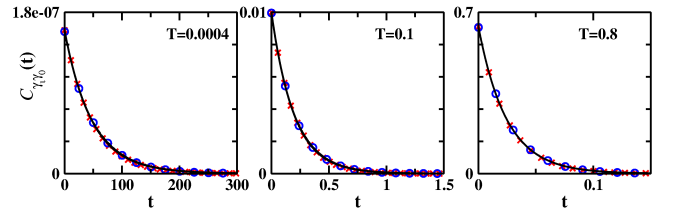

Figure 2: Same as Fig. 1 but for an inelastic gamma channel, $\mathcal{C}_{\gamma_i\gamma_0}(t)$ with $i \neq 0$.

When the neutron channel is closed, $\mathcal{C}_{b\gamma_0}(t)$ is very well approximated by an exponential, as expected. This is shown in Figs. 1 and 2 with the exponential fit shown as blue open circles but applies equally to all other cases calculated. The exponential is the same for all gamma channels. This is important because the signal is expected to be very weak for every single gamma channel. Summation over many such channels does not affect the form of the exponential. Measurements of a restricted sum $\sum_i \mathcal{C}_{\gamma_i\gamma_0}(t)$ would yield the average gamma width of the CN resonances.

Taken by itself, exponential decay is expected and not very exciting. The situation changes when the neutron channel is open. This is shown in Figs. 3 and 4. Because of the comparatively large value of T_n the neutron yield is significantly larger than the yield in any single gamma channel. Moreover, the time decay function differs significantly from an exponential, both in the neutron and in the gamma channels. As indicated in the captions, the curves
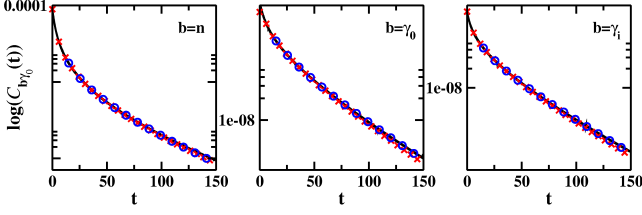
Figure 3: Logarithmic plot of $\mathcal{C}_{b\gamma_0}(t)$ versus t (in units of h/d) for an open neutron channel and $\Lambda = 49$ inelastic gamma channels, with $p_{\gamma_i} = p_{\gamma_0} = T$, $i = 1,...,49$, and values of the transmission coefficients $T = 0.0004$, $T_n = 0.2$ and of the final channel b as indicated in the insets. Solid line (color online: black): Eq. (16) for large times t. Crosses (color online: red): Approximation Eq. (20). Open circles (color online: blue): Fit of $t^{a_1} \exp(-a_2 t)$ to data. In all three cases $a_2 \simeq 0.02$ which is $\approx 50 \times T$. For $b = n$ we find $a_1 \simeq 1.46 \approx 3/2$, for $b = \gamma_0$, γ_i we have $a_1 \simeq 0.48 \approx 1/2$.

were fitted with a function of the form $t^{a_1} \exp(-a_2 t)$ (blue open circles). The best–fit value of a_2 is approximately equal to the sum of the transmission coefficients of the gamma channels. The exponent a_1 agrees approximately with the result given at the end of the last Section. Both results are in agreement with our expectations.
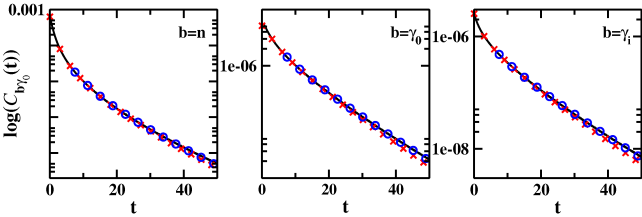

Figure 4: Same as Fig. 3 but for a different set of transmission coefficients $T_n = 0.4$, $T = 0.0016$. In all three cases $a_2 \simeq 0.077$ which is $\approx 50 \times T$. For $b = n$ we have $a_1 \simeq 1.39 \approx 3/2$, for $b = \gamma_0$, γ_i we obtain $a_1 \simeq 0.51 \approx 1/2$.

6. Summary and Conclusions

In nuclear reactions induced by short laser pulses of several MeV energy, the observable of interest is the time–decay function of the CN resonances. Provided the length Δt of the laser pulse is chosen optimally, the time–decay function is given by the Fourier transform of the S–matrix autocorrelation function. For that to be true, Δt must be large compared to the Heisenberg time $\hbar/d$ and small compared to the width of the giant dipole resonance. This fixes Δt to values between 0.5×10^{-19} s and 0.5×10^{-20} s, depending on mass number. The time–decay function comprises information on amplitude correlations of CN resonances which cannot be obtained from other observables.

We have calculated the time–decay function under the assumption that the laser–excited CN resonances are described by random–matrix theory. Our Eq. (20) gives an excellent approximation to that function. It depends on a small number of parameters only and is useful for the analysis of data. We have shown how to estimate these parameters from existing data (transmission coefficient for neutrons, average cross section for dipole absorption). The time–decay function rises steeply with time and reaches a maximum a short time after the initial laser pulse has hit the target. That time is of the order of the Heisenberg time. The further development in time of the time–decay function depends on whether the neutron channel is closed or open. In the first case, the time–decay function decreases exponentially. The decay width is given by the average total gamma decay width of the CN resonances and can be determined from data. In the second case, the time–decay function can be fit by the product of an exponential (again determined by the average total width for gamma decay) and a power law. The exponent of the latter depends on whether the final channel is the neutron channel or a gamma channel.

An experimental confirmation of our predictions would establish a new unambiguous test of random–matrix theory in nuclei. In addition, it would make it possible to measure the average total width for gamma decay of CN resonances located below neutron threshold.

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